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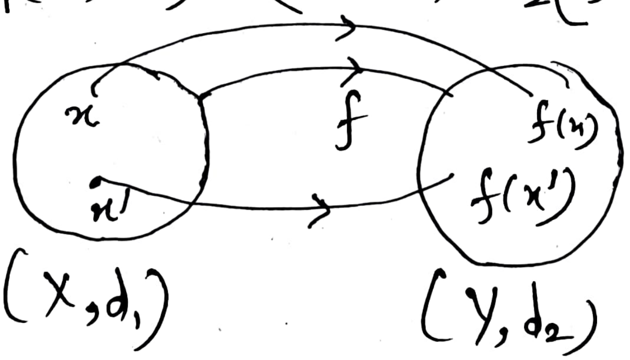
Uniform continuity

Let (X, d_1) and (Y, d_2) be two metric spaces then a mapping f of X into Y i.e.

$f: X \to Y$ is said to be uniformly continuous if for each $\epsilon > 0$, there exists $\delta > 0$ such that

$$d_1(x, x') < \delta \Rightarrow d_2(f(x), f(x')) < \epsilon$$



⇒ It is obvious that any uniformly continuous mapping is necessarily continuous.

But continuous mapping does not necessarily imply uniformly continuous mapping.

For example, the function f defined on the set R of real numbers by

$f(x) = x^2$ is continuous but not uniformly continuous. Similarly $f(x) = \frac{1}{x}$ defined on $(0, 1)$ is continuous but not uniformly continuous.